Correction to the quantum relation of photons involved in the Doppler effect in the framework of a special Lorentz violation model

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Abstract

The possibility of Lorentz symmetry breaking has been discussed in many models of quantum gravity. In this paper we followed the Lorentz violation model in Ref. [1] to discuss the Doppler frequency shift of photons and the Compton scattering process between photons and electrons, pointing out that following the idea in Ref. [1] we have to modify the usual quantum relation of photons involved in the Doppler effect. But due to the current limited information and knowledge, we couldn't yet determine the specific expression for the correction coefficient of the quantum relation of photons. However, the phenomenon of spontaneous radiation in a cyclotron maser give us an opportunity to see what the expression for this correction coefficient might look like, as the phenomenon of spontaneous radiation in a cyclotron maser can be explained by the Doppler effect of virtual photons and the Compton scattering process between virtual photons and electrons (or other particles). Therefore, under some restrictive conditions, we construct a very concise expression for this correction coefficient by discussing different cases. And then we used this expression to analyze the wavelength of radiation in the cyclotron maser, which tends to be a limited value at $\boldsymbol{v} \rightarrow \boldsymbol{c}$, rather than to be 0 as predicted by the Lorentz model. And we also discussed the inverse Compton scattering phenomenon and found that there is a limit to the maximum energy that can be obtained by photons in the collision between ultra-high energy particles and low-energy photons, which conclusion is also very different from that obtained from the Lorentz model, in which the energy that can be obtained by the photon tends to be infinite as the velocity of particle is close to $\boldsymbol{c}$. This paper still inherits the idea in Ref. [1] that the energy and momentum of particles (i.e., any particles, including photons) cannot be infinite, otherwise it will make some physical scenarios meaningless, and this view is also from the idea in some quantum gravity models. When the parameter Q characterizing the degree of deviation from the Lorentz model is equal to 0 , all the results and conclusions in this paper will return to the case as in the Lorentz model, so this paper also provides us with a possible experimental scheme to determine the value of Q in Ref. [1], although it still requires extremely high experimental energy.

Keywords: Lorentz model, the speed of light, Lorentz violation, Doppler effect, Compton scattering, inverse Compton scattering, quantum relation

1. Introduction

The study of quantum gravity has always been a frontier and hot topic in fundamental physics, the main reason is that people find that black holes have singularity problem, information loss

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problem, and also find that general relativity and quantum theory are incompatible under the current framework [2–5]. The starting point for many quantum gravity models is the belief that Lorentz symmetry may be violated at ultra-high energy scale, such as near the Planck energy scale. The most typical model to hold this view is the double special relativity model (DSR), which not only considers the speed of light to be a constant, but also introduces another constant called "minimum length or maximum energy", which leads to a modification of the energy-momentum dispersion relationship of particles at extremely high energy scale [6, 7]. The research on the DSR model can be seen in many literature, and which is believed to be able to solve some difficult problems in the ultra-high energy field [8–13].

However, the DSR model has an obvious disadvantage that it is not concise enough. The equation describing the dispersion relationship of particles usually contains many parameters, for example, in the DSR model, the dispersion relationship of particles at Planck energy scale is usually expressed as $E^2 = \boldsymbol{p}^2 + m^2 + \eta L_p^\alpha \boldsymbol{p}^2 E^\alpha + O\left(L_p^{\alpha+1} E^{\alpha+3}\right)$, where L_p denotes the "Planck length", α is a positive integer and η is a real number [8]. Since the physical meaning of these parameters is not clear [14], many researchers only take one or two of these parameters to study. For this reason, Ref. [1] proposed another possible Lorentz violation model, which contains only one parameter in the equation describing the dispersion relationship of particles. More importantly, the Lorentz violation model proposed in Ref. [1] naturally returns to the Lorentz model at low and medium energy scale, and when the velocity of particle is close to $\boldsymbol{c}$ the energy of the particle tends to be a limited value (which is similar to the DSR model), rather than to be infinite as predicted by the Lorentz model. In order to clarify the purpose and viewpoint of this paper, we can first briefly review the Lorentz violation model proposed in Ref. [1].

In Ref. [1] it assumed that the speed of light observed by an observer moving with velocity $\boldsymbol{v}$ relative to the light source is $n\boldsymbol{c}$, where n is variable or invariable value independent of the direction of $\boldsymbol{v}$ and $\boldsymbol{c}$. and $n(\boldsymbol{v}=0,\boldsymbol{c}) = 1$. Thus based on this assumption the coordinate transformation between two inertial systems moving with each other at a relative velocity $\boldsymbol{v}$ is

$$\left\{\begin{array}{l} \boldsymbol{x}' = \gamma(\boldsymbol{x} - \boldsymbol{v}t) \\ t' = \gamma\left(t - \frac{\boldsymbol{v}}{k^2(\boldsymbol{v},\boldsymbol{c})}\boldsymbol{x}\right) \end{array}\right. \tag{1}$$

where $\gamma(\boldsymbol{v},\boldsymbol{c}) = 1/\sqrt{1-\boldsymbol{v}^2/k^2}$, $\quad k(\boldsymbol{v},\boldsymbol{c}) = \sqrt{n\boldsymbol{v}\boldsymbol{c}^2/(n\boldsymbol{c}-\boldsymbol{c}+\boldsymbol{v})}$. And the value of n is independent of the direction of $\boldsymbol{v}$ and $\boldsymbol{c}$.

From Eq. (1) one can obtain that if $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = \boldsymbol{c}$, then $\mathrm{d}\boldsymbol{x}'/\mathrm{d}t' = n\boldsymbol{c}$, and if $\mathrm{d}\boldsymbol{x}'/\mathrm{d}t' = \boldsymbol{c}$, then $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = n\boldsymbol{c}$, which ensures that the two inertial systems are equivalent.

Eq. (1) is similar in form to the Lorentz transformation, and it has been shown in Ref. [1] that Maxwell's equations are also covariant based on Eq. (1). And at the same time, correspondingly, the dispersion relation of particle with rest mass m_0, compared to the form as in the Lorentz model, is modified as

$$E^2 = \boldsymbol{p}^2 k^2 + E_0^2 \tag{2}$$

Where $E_0 = m_0 k^2, E = \gamma m_0 k^2$ denotes the particle's total energy, $\boldsymbol{p} = \gamma m_0 v$ denotes the particle's momentum, and $\gamma(\boldsymbol{v},\boldsymbol{c}) = 1/\operatorname{sqrt}\left(1-\boldsymbol{v}^2/k^2\right), k(\boldsymbol{v},\boldsymbol{c}) = \operatorname{sqrt}\left[n\boldsymbol{v}\boldsymbol{c}^2/(n\boldsymbol{c}-\boldsymbol{c}+\boldsymbol{v})\right]$.

Next, some people may easily find the problem that if the dispersion relation of particles is modified as Eq. (2), then the usual quantum relation of photons (see Eq. (3)) should also need to be modified, correspondingly. The reason is that for massless particle, the dispersion relation is $E = \boldsymbol{p}k$ rather than $E = \boldsymbol{p}\boldsymbol{c}$ based on Eq. (2). Secondly, in Ref. [1] it allows the speed of

light is variable between inertial systems (especially it allows the observed speed of light tends to be 0 when the speed of light source relative to the observer is close to $\boldsymbol{c}$), so the usual quantum relation of photons (i.e., Eq. (3)) will no longer be applicable in this case.

$$\begin{cases} E = hf \\ \boldsymbol{p} = \frac{h}{\lambda} \end{cases} \tag{3}$$

Where h is the Planck constant, f and λ denotes the frequency and wavelength of light, respectively.

Therefore, this is the issue to be discussed in this paper. However, one may wonder why we need to modify the quantum theory when it has been so successful so far. The answer is that our current quantum theory is based on Lorentz symmetry, and if the energy is high enough that the Lorentz symmetry is obviously broken, the quantum theory should be modified. One piece of evidence is the proposal of various models of quantum gravity, which aim to explore the new physics that may emerge when energy is pushed to a very high level.

However, when we try to construct or find such a quantum relation of photon that satisfies Eq. (2), we found that it is very difficult to determine the specific expression, for that the set of expression satisfying $E = \boldsymbol{p}k$ is large. But fortunately, a physical phenomenon in the cyclotron maser called synchrotron radiation can give us a glimpse of what this expression might look like.

In Ref. [15–17] the author discussed the phenomenon of spontaneous radiation and synchrotron radiation in a cyclotron maser using the concept of virtual photons. By considering the Doppler effect and Compton scattering effect of virtual photons, the author obtained the formulas of spontaneous radiation and synchrotron radiation in the cyclotron maser, which are same as the classical theoretical results. Inspired by the idea in Ref. [15–17], in this paper we first discussed the restrictive conditions for the modified quantum relation of photons, and then based on which we try to find a possible correction expression for the quantum relation of photons that satisfies Eq. (2). And further discussed the effect of this modified quantum relation of photon on synchrotron radiation in the cyclotron maser. This paper was thus organized as follows. In Sect. 2, we discussed the Doppler effect of photons based on Ref. [1], and in Sect. 3 we discussed the corresponding Compton scattering effect of photons. In order to apply the Doppler effect and Compton effect of photons to a physical phenomenon at the same time, we choose to analyze the spontaneous and synchrotron radiation phenomena in a cyclotron maser, which is shown in Sect. 4, and at the same time we discussed the possible correction coefficient of the quantum relation of photons. In Sect. 5 we discussed the inverse Compton scattering phenomenon and pointed out the difference in predicted results between the model in Ref. [1] and the Lorentz model. Finally we summarized the paper.

2. Doppler effect of photons

As we know that the frequency of light emitted by a light source is different when measured in a moving inertial system relative to the light source, which is called the Doppler frequency shift effect. As shown in Figure 1, if the inertial system S' moves along the $\boldsymbol{x}$ ($\boldsymbol{x}'$)-axis with velocity $\boldsymbol{v}$ relative to the inertial system S, and the frequency and wavelength of light emitted by the light source fixed in S' is f_0 and λ_0, respectively. At the same time we assume the angle is θ between the direction of light propagation and the $\boldsymbol{x}'$-axis. Then based on the conclusion in Ref. [1], for

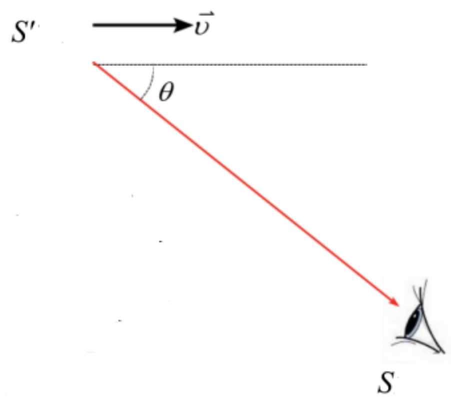


Figure 1: Doppler effect for moving light source.

the observer in S the clock period of the light source will slow down as

$$T = \frac{T_0}{\sqrt{1-\beta^2}} = \gamma T_0 \tag{4}$$

Where $\beta = \boldsymbol{v}/k, T_0$ and T represent the clock period measured in S' and S, respectively. And $T_0 = 1/f_0$, $\lambda_0 f_0 = \boldsymbol{c}$.

On the other hand, as the light source is moving relative to the observer, the observed wavelength of light in S is

$$\lambda = n\boldsymbol{c}T - (\boldsymbol{v}\cos\theta)T \tag{5}$$

Where $n = n(\boldsymbol{v}\cos\theta, \boldsymbol{c})$.

From Eq. (4) and (5), we can obtain

$$\begin{cases} f = \frac{n\boldsymbol{c}}{\lambda} = \frac{\sqrt{1-\beta^2}}{1-\frac{\boldsymbol{v}\cos\theta}{n\boldsymbol{c}}} f_0 \\ \lambda = n\left(1 - \frac{\boldsymbol{v}\cos\theta}{n\boldsymbol{c}}\right)\frac{\lambda_0}{\sqrt{1-\beta^2}} \end{cases} \tag{6}$$

Where f and λ is the frequency and wavelength of light measured in S, respectively.

Eq. (6) shows the corresponding Doppler effect based on the conclusion in Ref. [1], and it can be seen that when $n \equiv 1$ Eq. (6) returns to the usual case as in the Lorentz model.

Based on Eq. (6) we next discuss a special case where $\theta = 0$ (if $\theta = \pi/2$, then $n(0, \boldsymbol{c}) = 1$, Eq. (6) returns to the case as in the Lorentz model), in this case one can obtain that

$$\begin{cases} f = \frac{n\boldsymbol{c}}{\lambda} = \frac{\sqrt{1-\beta^2}}{1-\frac{\boldsymbol{v}}{n\boldsymbol{c}}} f_0 \\ \lambda = n\boldsymbol{c}\left(1 - \frac{\boldsymbol{v}}{n\boldsymbol{c}}\right)\frac{\lambda_0}{\sqrt{1-\beta^2}} \end{cases} \tag{7}$$

Here it should be noted that $n = n(\boldsymbol{v}, \boldsymbol{c})$ in Eq. (7) due to $\boldsymbol{v}\cos\theta = \boldsymbol{v}$.

Based on Eq. (7), one may notice that when $\boldsymbol{v} = n\boldsymbol{c}, \lambda = 0$ and f tends to be infinite, which is similar to the property of shock wave in acoustic Doppler effect. From Ref. [1] we can obtain the condition satisfying the shock wave of light, that is, according to Ref. [1], when $\boldsymbol{v} \sim \boldsymbol{c}$, there is

$$n = \frac{1}{1-Q}\left[1 - Q^{1-\left(\frac{v}{c}\right)^2}\right] = \frac{1}{1-Q}\left[1 - Q^{\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}\right] \approx \frac{1}{1-Q}\left[1 - Q^{2\left(1-\frac{v}{c}\right)}\right] \approx \frac{2\ln Q}{Q-1}\left(1 - \frac{\boldsymbol{v}}{\boldsymbol{c}}\right) \tag{8}$$

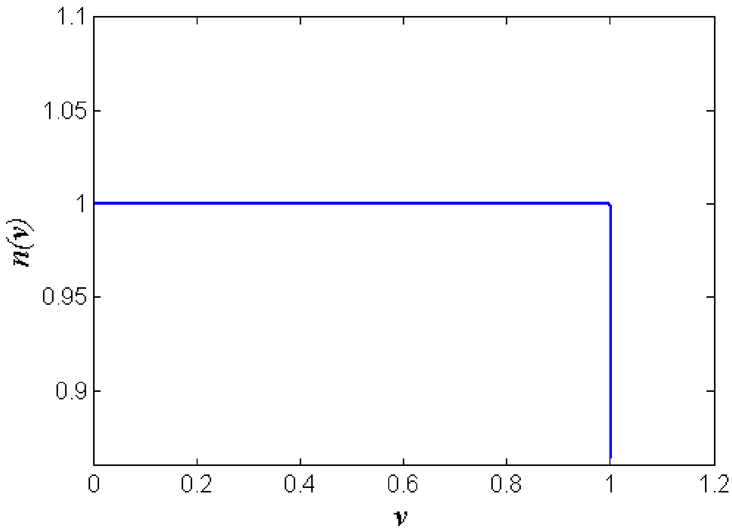


Figure 2: The curve of $n(\boldsymbol{v}) \sim \boldsymbol{v}$ when taking $Q = (1/e)^{10^6}$ (set $\boldsymbol{c} = 1$). Since here it shows the global picture of the curve, a transition arc at the corner is too small to show [1].

Where Q is a constant and its value needs to be determined by the experiment. As mentioned in Ref. [1], we do not yet know the specific value of Q because the current experimental energy is not enough, but it can be known from a large number of existing experimental results that $Q \sim 0$.

As already explained in Ref. [1] that the reason why n is chosen as the form in Eq. (8) is that the function of Eq. (8) can be well consistent with various experimental results conducted at low or medium energy scale at present. And also importantly, Eq. (8) is very concise for it has only one parameter Q. Here let's re-show the curve of $n \sim \boldsymbol{v}$ as Figure 2.

n can be viewed as the degree to which the speed of light observed by the observer in a moving inertial systems relative to the light source deviates from $\boldsymbol{c}$. As can be seen from Figure 2, n is almost equal to 1 in large range of $\boldsymbol{v}$. And it's this property of n that's why it fits well with various experimental results conducted at low or medium energy scale at present. When $Q = 0$, $n \equiv 1$, and correspondingly, Eq. (1) and Eq. (2) return to the case as in the Lorentz model.

So for $\boldsymbol{v} = n\boldsymbol{c}$, there is

$$\boldsymbol{v}_{\text{wave shock}} \approx \frac{1}{\frac{Q-1}{2\ln Q} + 1}\boldsymbol{c} < \boldsymbol{c} \tag{9}$$

Where $\boldsymbol{v}_{\text{wave shock}}$ is the solution for $\boldsymbol{v} = n\boldsymbol{c}$ and Eq. (9) is just the condition for generating a shock wave of light.

Similarly, we can express the Mach cone of light as in Figure 3, and where α is the angle of cone.

In Figure 3 the spatial distribution of the speed of light along the center of the light source satisfies the following curve equation $\rho = n(\boldsymbol{v}\cos\theta, \boldsymbol{c})c$ in polar coordinates. When $\boldsymbol{v} > \boldsymbol{v}_{\text{wave shock, a}}$ reverse timing sequence will occur. But as we know, the timing sequence can be reversed in independent events without violating the law of causality (but not in dependent events).

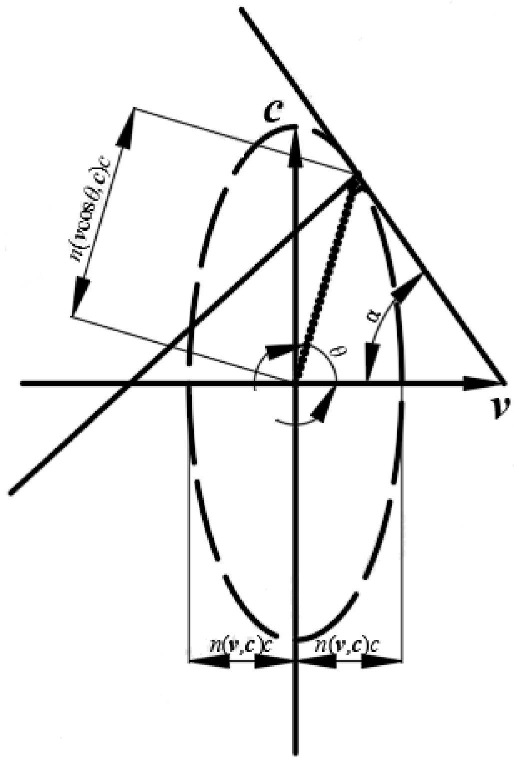


Figure 3: Geometric diagram of the Mach cone of light.

3. Compton scattering

We know from the Compton scattering process that light can be regarded as a particle (i.e., photon), in the theory of special relativity, the energy and momentum of photons can be expressed by Eq. (3). However, if we substitute Eq. (7) into Eq. (3), we can obtain

$$\frac{E}{\boldsymbol{p}} = n\boldsymbol{c} \tag{10}$$

Eq. (10) does not agree formally with Eq. (2), for that based on Eq. (2), for massless particles there is $E = \boldsymbol{p}k$. So we should modify the quantum relation of photons if we followed the idea in Ref. [1]. Here we assume the modified quantum relation as

$$\begin{cases} E = a_1 hf \\ \boldsymbol{p} = a_2 \frac{h}{\lambda} \end{cases} \tag{11}$$

Then based on $E = \boldsymbol{p}k$ we have $a_1/a_2 = k/n$. But unfortunately, there doesn't seem to be any information or knowledge to be used to help us to specify the expression for a_1 and a_2.

Next we attempt to discuss the Compton scattering process between photons and electrons based on Eq. (11), and the corresponding diagram can be seen in Figure 4. Here for simplicity, we assume the electrons are stationary relative to the laboratory before the collision occurs.

According to the diagram of interaction between photon and electron in Figure 4, we can set

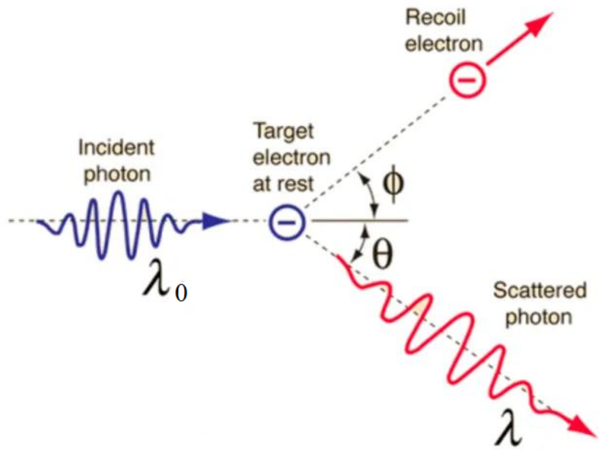


Figure 4: The diagram of Compton scattering.

out the following energy conservation and momentum conservation equations

$$\begin{cases} a_1 h f_0 + m_0 \boldsymbol{c}^2 = a_1' h f + m k^2 \\ a_2 \frac{h}{\lambda_0} = a_2' \frac{h}{\lambda} \cos\theta + m \boldsymbol{v} \cos\phi \\ a_2' \frac{h}{\lambda} \sin\theta = m \boldsymbol{v} \sin\phi \\ \frac{a_1}{a_2} = \frac{k}{n} \\ \frac{a_1'}{a_2'} = \frac{k'}{n'} \end{cases} \tag{12}$$

Note that in Eq. (12) $a_1, a_2, k, n, a_1', a_2', k', n'$ are both functions of $\boldsymbol{v}$ and $\boldsymbol{c}$. And Eq. (12) can be approximately simplified, that is, when $\boldsymbol{v} \ll \boldsymbol{c}, k \approx c$, and when $\boldsymbol{v} \sim \boldsymbol{c}$, there is

$$\lim_{\boldsymbol{v}\to\boldsymbol{c}} k = \lim_{\boldsymbol{v}\to\boldsymbol{c}} \sqrt{\frac{n\boldsymbol{v}\boldsymbol{c}^2}{n\boldsymbol{c} - \boldsymbol{c} + \boldsymbol{v}}} = \lim_{\boldsymbol{v}\to\boldsymbol{c}} \sqrt{\frac{\boldsymbol{v}\boldsymbol{c}^2}{\boldsymbol{c} - \frac{\boldsymbol{c}-\boldsymbol{v}}{n}}} = \sqrt{\frac{\boldsymbol{c}^2}{\boldsymbol{1} - \frac{Q-1}{2\ln Q}}} \tag{13}$$

Due to $Q \sim 0$, then based on Eq. (13), when $\boldsymbol{v} \sim c$, there is $k \sim c$. That is, for $\boldsymbol{v} \in [0, \boldsymbol{c})$, there is $k \approx c$. So we can assume $k \approx k' \approx c$ all the time for Eq. (12). Then based on Eq. (12) we can obtain

$$\frac{\lambda}{a_2'} - \frac{\lambda_0}{a_2} = \frac{2h}{m_0 c^2}(1-\cos\theta) \frac{k\left(\frac{1}{\sqrt{1-\beta^2}} - \frac{c^2}{k^2}\right)}{\frac{2}{\sqrt{1-\beta^2}} - \frac{c^2}{k^2} - \frac{k^2}{c^2}} \approx \frac{h}{m_0 c}(1-\cos\theta) \tag{14}$$

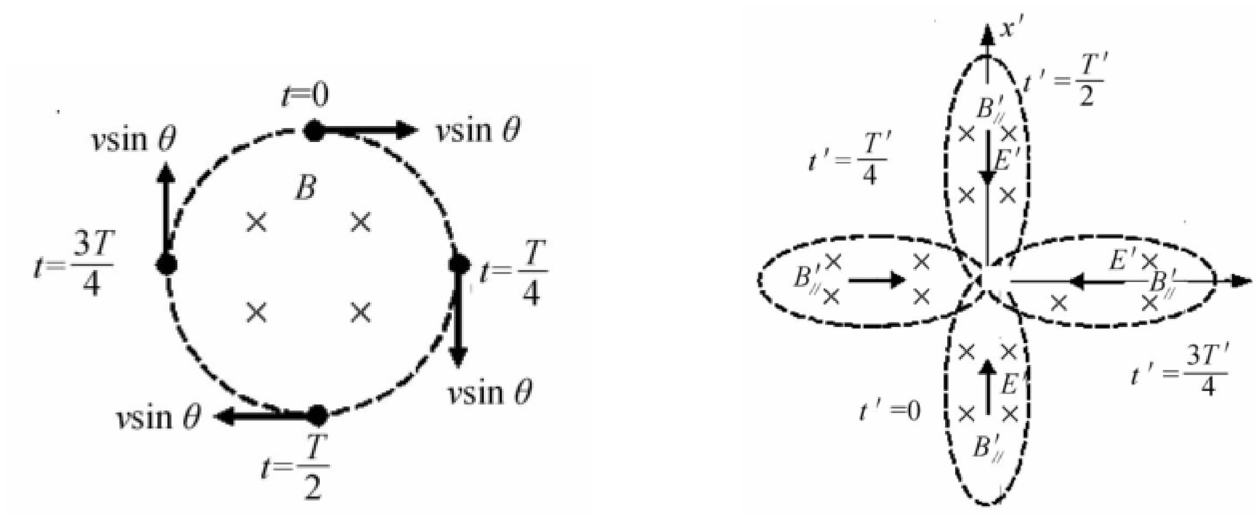
where m_0 is the mass of electron.

Above we obtained the formula of Doppler frequency shift and Compton scattering satisfying Eq. (2), but we do not yet know the specific expressions for a_1 and a_2. What we can do next is put the two formulas into real physical situations to see what the expressions for a_1 and a_2 might look like. And fortunately, the phenomenon of spontaneous and synchrotron radiation in the cyclotron maser just provide such a physical application.

4. Spontaneous radiation in cyclotron maser

Cyclotron maser is a kind of device with high peak power and high average power in millimeter band and sub-millimeter band. After several decades of efforts, the mechanism, experiment design

and numerical simulation subjecting to the cyclotron maser have been analyzed extensively [18–23]. For example, in Ref. [15–17], the mechanism of Compton scattering of virtual photons is used to analyze the spontaneous radiation in a cyclotron maser (In the theory of spontaneous radiation for the free electron laser, the periodic static magnetic field in the oscillator is also regarded as virtual photons, which are then converted into real photons by Compton scattering with high-energy particles [24]). Inspired by this idea, now we will revisit the discussion on spontaneous radiation in a cyclotron maser using Eq. (6) and Eq. (14).



(a) The laboratory system

(b) The particle system

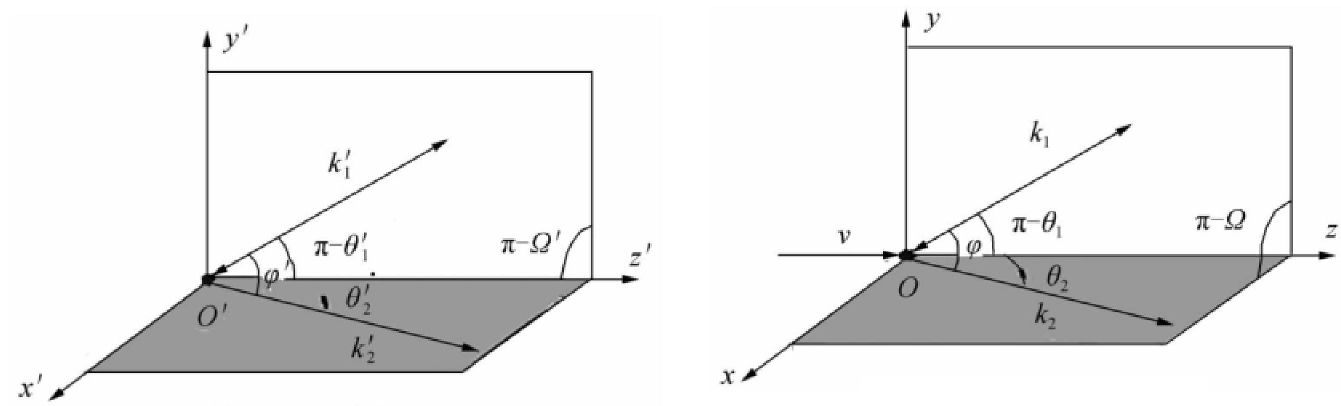
Figure 5: The relative motion between magnetic fields and charged particles [15].

In the laboratory system, we set the charged particle have a charge of q, a rest mass of m_0, a velocity of $\boldsymbol{v}$ perpendicular to the magnetic field $\boldsymbol{B}$. Then in the laboratory system, the spiral motion of the charged particle in a magnetic field has a period of

$$T_c = \frac{2\pi R}{\boldsymbol{v}} = \frac{2\pi m_0}{Bq\sqrt{1-\beta^2}} \tag{15}$$

As shown in Figure 5b), we construct an inertial systems S' that is stationary relative to the particle and the direction of v is parallel to the z' axis. Based on the relativistic transformation, there will be both magnetic and electric fields in S'. And based on the “moving ruler” effect, in S' the track perimeter of the magnetic field in reverse spiral motion around the charged particle becomes $2\pi R/\gamma$, then correspondingly, its cycle period is

$$T'_c = \frac{2\pi R/\gamma}{\boldsymbol{v}} = T_c/\gamma \tag{16}$$



(a) The particle system S^n (b) The laboratory system

Figure 6: Angle definition in Compton scattering.

According to Ref. [15], since the photon in Eq. (16) cannot be observed in the laboratory system, but can only be observed in S' due to the relativistic effects, the photon can be regarded as virtual photon. Next, we will consider the Doppler effect of virtual photon as the electric and magnetic fields that vary periodically both move at a velocity $-\boldsymbol{v}$ relative to S'. Now first we define the angle shown in Figure 6, that is, in the particle system $S', \theta_1{}'$ denotes the angle between the direction of virtual photon propagation and the z' axis, θ_2' denotes the angle between the direction of the scattered photon propagation and the z' axis, φ' denotes the scattering angle. Ω' is the angle between the plane formed by the direction of virtual photon propagation and the z' axis and the plane formed by the direction of scattered photon propagation and the z' axis, and in the laboratory system, θ_1 denotes the angle between the direction of virtual photon propagation and the $\boldsymbol{z}$ axis, θ_2 denotes the angle between the direction of the scattered photon propagation and the z axis, φ denotes the scattering angle. Ω is the angle between the plane formed by the direction of virtual photon propagation and the z axis and the plane formed by the direction of scattered photon propagation and the z axis. Therefore, based on Eq. (6) we can obtain the wavelength of virtual photons in the particle system S' as the source of the virtual photon is moving at a velocity $-\boldsymbol{v}$ relative to the particle system (setting $c = 1$)

$$\lambda_1' = \frac{\lambda_c'}{\sqrt{1-\beta^2}} n \left[1 - \frac{\boldsymbol{v}}{n} \cos\left(\pi - \theta_1'\right)\right] \tag{17}$$

where $\lambda_c{}' = \boldsymbol{c}T_c{}' = \boldsymbol{c}T_c/\gamma$ corresponding to Eq. (4) and $n = n\left(\boldsymbol{v}\cos\left(\pi - \theta_1'\right), \boldsymbol{c}\right)$.

Virtual photons can be scattered and radiated out as real photons, and based on Eq. (14) the wavelength of radiated photons satisfies

$$\frac{\lambda_2'}{a_2'} - \frac{\lambda_1'}{a_2} \approx \frac{h}{m_0}\left(1 - \cos\varphi'\right) \tag{18}$$

where λ_2' denotes the wavelength of radiated photons.

Again, using the Doppler formula in Eq. (6), we can obtain the wavelength of radiated photons observed by the observer in the laboratory system

$$\lambda_2 = \frac{\lambda_2'}{\sqrt{1-\beta^2}} n' \Big[1 - \frac{v}{n'} \cos\theta_2\Big)\Big] \tag{19}$$

where $n' = n\left(\boldsymbol{v}\cos\theta_2, \boldsymbol{c}\right)$.

Since $Q \sim 0, a_1, a_2$ in Eq. (11) should have $a_1 \approx 1, a_2 \approx 1$ at low or medium frequency shift (That means that the light source is not moving very fast). And on the other hand, since the Compton wavelength of the charged particle is generally much smaller than $\boldsymbol{c}T_c$, then we can assume $a_2' \approx 1$ in Eq. (18). So based on Eq. (14)∼(19), we can obtain

$$\lambda_2 = \frac{n' - \boldsymbol{v}\cos\theta_2}{\sqrt{1-\beta^2}}\left[T_c\frac{n}{a_2}\left(1+\frac{\boldsymbol{v}}{n}\cos\theta_1'\right)+\frac{h}{m_0}\left(1-\cos\varphi'\right)\right] \tag{20}$$

As stated in Ref. [1] that, the formal difference between the proposed model in Ref. [1] and the Lorentz model is that replacing c in the Lorentz model by k can obtain the model in Ref. [1]. So the formula of aberration of light here can be obtained

$$\begin{cases} \cos\theta' = \frac{\cos\theta-\beta}{1-\beta\cos\theta} \\ \sin\theta' = \frac{\sin\theta}{\gamma(1-\beta\cos\theta)} \end{cases} \tag{21}$$

Where $\theta = \theta_1$ or $\theta = \theta_2$ or $\theta = \Omega$.

On the other hand, according to the geometric definition in Figure 6, one can obtain the following geometric relationship

$$\begin{aligned} \cos\varphi' &= \cos\theta_1'\cos\theta_2' + \sin\theta_1'\sin\theta_2'\cos\Omega' \\ \cos\varphi &= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\Omega \end{aligned} \tag{22}$$

Substituting Eq. (21) and (22) into Eq. (20), then

$$\lambda_2 = \frac{n-\boldsymbol{v}\cos\theta_2}{\sqrt{1-\beta^2}}\left[T_c\frac{n}{a_2}\left(1+\frac{k}{n}\frac{\beta\cos\theta_1-\beta^2}{1-\beta\cos\theta_1}\right)\right]+\frac{h}{m_0}\sqrt{1-\beta^2}\left(n-\boldsymbol{v}\cos\theta_2\right)\frac{1-\cos\varphi}{\left(1-\beta\cos\theta_1\right)\left(1-\beta\cos\theta_2\right)} \tag{23}$$

Eq. (23) is just the wavelength of spontaneous radiation of a charged particle in a Cyclotron Maser that is observed by the observer in the laboratory system. At low or medium frequency shift, there has $n \approx 1, a_2 \approx 1, k \approx 1$, then Eq. (23) returns to the following equation

$$\lambda_2 = T_c\sqrt{1-\beta^2}\frac{1-\beta\cos\theta_2}{1-\beta\cos\theta_1}+\frac{h}{m_0}\sqrt{1-\beta^2}\frac{1-\cos\varphi}{1-\beta\cos\theta_1} \tag{24}$$

Which corresponds to case as in the Lorentz model.

But here we're more interested in what happens at ultra-high frequency shift. First it reminds us that due to it allowed $\boldsymbol{v} \in [0, \boldsymbol{c})$ in Ref. [1], when $\boldsymbol{v} = n\boldsymbol{c}$ the energy of the scattered virtual photon in Eq. (12) maybe infinite because of $\lambda_1' = 0$ unless we assume

$$\begin{cases} a_1 = \frac{1-\frac{\boldsymbol{V}}{n\boldsymbol{c}}}{1-n\frac{\boldsymbol{V}}{\boldsymbol{c}}} \\ a_2 = \frac{n}{k}a_1 \approx na_1 = n\frac{1-\frac{\boldsymbol{V}}{n\boldsymbol{c}}}{1-n\frac{\boldsymbol{V}}{\boldsymbol{c}}} \end{cases} \tag{25}$$

Where $\boldsymbol{V} = \boldsymbol{v}\cos\theta, n = n(\boldsymbol{V}, \boldsymbol{c}), \theta$ is the angle of direction between $\boldsymbol{v}$ and $\boldsymbol{c}$.

Eq. (25) is motivated by the following considerations

1) Since the expression of λ in Eq. (6) have the term $[1-\boldsymbol{v}\cos\theta/(n\boldsymbol{c})]$, which let $\lambda=0$ under the condition of $\boldsymbol{v}=n\boldsymbol{c}$ and $\theta=0$, and $\lambda=0$ will make the energy of the virtual photon to be infinite, which will make Eq. (14) invalid. So in order to avoid that, we hope a_2 in Eq. (11) have the term $[1-\boldsymbol{v}\cos\theta/(n\boldsymbol{c})]$ to cancel out this same term in λ.

2) On the other hand, we also want to extend Eq. (23) to the case where $\boldsymbol{v}\to\boldsymbol{c}$, and in this case, according to Ref. [1] we know that

$$\left\{\lim_{\boldsymbol{v}\to\boldsymbol{c}}\frac{1-\boldsymbol{v}/\boldsymbol{c}}{n}=\frac{Q-1}{2\ln Q}\right. \tag{26}$$

Eq. (26) reminds us that in order to avoid in the case where $\boldsymbol{v}\to\boldsymbol{c}, a_2$ is infinite, we can replace $\boldsymbol{v}/\boldsymbol{c}$ by $n\boldsymbol{v}/\boldsymbol{c}$ for a_2, then for $\boldsymbol{v}\to\boldsymbol{c}$, the expression of $1/(1-n\boldsymbol{v}/\boldsymbol{c})$ tends to 1 instead of the expression of $1/(1-\boldsymbol{v}/\boldsymbol{c})$ tends to be infinite, which argument seems a little hard to understand here, we can see how it works in the following

Substituting Eq. (25) and Eq. (6) into Eq. (11), one will obtain

$$\begin{cases} E=a_1hf=\frac{\sqrt{1-\beta^2}}{1-\frac{n\boldsymbol{V}}{\boldsymbol{c}}}hf_0 \\ \boldsymbol{p}=a_2\frac{h}{\lambda}=\frac{E}{k}\approx E \end{cases} \tag{27}$$

Based on Eq. (27), it has

a) $\lim_{V\to c}E=a_1hf=\sqrt{1-\beta^2}/(1-n\boldsymbol{V})hf_0=\sqrt{1-\beta^2}hf_0=\sqrt{(Q-1)/(2\ln Q)}hf_0$ and $\boldsymbol{p}=E/k\approx E$.

b) $\lim_{V\to -c}E=a_1hf=\sqrt{1-\beta^2}/(1+n\boldsymbol{V})hf_0=\sqrt{1-\beta^2}hf_0=\sqrt{(Q-1)/(2\ln Q)}hf_0$ and $\boldsymbol{p}=E/k\approx E$.

In the Lorentz model, as we know that for the case where $\boldsymbol{V}\to\boldsymbol{c}, E$ tends to be infinite, however, the above we constrain E to be a limited value that even if in the case where $\boldsymbol{V}=n\boldsymbol{c}, f$ is infinite and $\lambda=0$, but $E=\sqrt{1-\beta^2}/\left(1-\boldsymbol{V}_{\text{wave shock}}{}^2\right)hf_0$ is also a limited value. It is the limited energy and momentum that makes Compton scattering meaningful.

3) In addition, it has been stated above that due to $Q\sim 0$ it should have $a_1\approx 1, a_2\approx 1$ at low or medium frequency shift, which means that it should return to the case as in the Lorentz model at low or medium frequency shift.

Based on the three key considerations above, we construct the expression for a_1 and a_2 as Eq. (25). One may consider other possible expressions for a_1 and a_2, but after many attempts we found that Eq. (25) is the most concise expression that satisfies the above three requirements. So here in this paper we will just try to use this expression to analyze the phenomenon of synchrotron radiation in the cyclotron maser.

Now we have known that Eq. (23) returns to Eq. (24) at low or medium frequency shift. Thus next we will discuss the case where $\boldsymbol{v}\to\boldsymbol{c}$, and in this case we can obtain $\theta_2\approx 1/\gamma\approx 0$, then Eq. (23) can be written accordingly as (setting $\boldsymbol{c}=1$)

$$\lambda_2=\frac{n'-\boldsymbol{v}}{\sqrt{1-\beta^2}}\left[T_c\frac{n}{a_2}\left(1+\frac{k}{n}\frac{\beta\cos\theta_1-\beta^2}{1-\beta\cos\theta_1}\right)\right]+\frac{h}{m_0}\sqrt{1-\beta^2}(n-\boldsymbol{v})\frac{1-\cos\varphi}{(1-\beta\cos\theta_1)(1-\beta)} \tag{28}$$

Where $n'=n\left(\boldsymbol{v}\cos\theta_2,\boldsymbol{c}\right)\approx n(\boldsymbol{v},\boldsymbol{c})$, and

$$a_2\approx n\frac{1-\frac{\boldsymbol{v}\cos\left(\pi-\theta_1'\right)}{n}}{1-n\boldsymbol{v}\cos\left(\pi-\theta_1'\right)}=n\frac{1+\frac{\boldsymbol{v}\cos\theta_1'}{n}}{1+n\boldsymbol{v}\cos\theta_1'}=n\frac{1+\frac{\boldsymbol{v}}{n}\frac{\cos\theta_1-\beta}{1-\beta\cos\theta_1}}{1+n\boldsymbol{v}\frac{\cos\theta_1-\beta}{1-\beta\cos\theta_1}} \tag{29}$$

When $\theta_1 = 0, \cos\theta_1' = (1-\beta)/(1-\beta) = 1$, then it means $\theta_1' = 0$, so $n = n\left(\boldsymbol{v}\cos\left(\pi - \theta_1'\right), \boldsymbol{c}\right) = n(-\boldsymbol{v}, \boldsymbol{c}) = n(\boldsymbol{v}, \boldsymbol{c})$ in Eq. (17). Then we can obtain

$$
\begin{aligned}
\lim_{\boldsymbol{v}\to 1}\lambda_2 &= \lim_{\boldsymbol{v}\to 1}\frac{n-\boldsymbol{v}}{\sqrt{1-\beta^2}}\left[T_c\frac{n(1+n\boldsymbol{v})}{n+\boldsymbol{v}}\left(1+\frac{1}{n}\beta\right)\right] + \frac{h}{m_0}\sqrt{1-\beta^2}(n-\boldsymbol{v})\frac{1-\cos\varphi}{(1-\beta)^2} \\
&= \lim_{\boldsymbol{v}\to 1}\frac{T_c}{\sqrt{1-\beta^2}}\frac{n(n-\boldsymbol{v})(1+n\boldsymbol{v})}{n+\boldsymbol{v}} + \frac{T_c}{\sqrt{1-\beta^2}}\frac{(n-\boldsymbol{v})(1+n\boldsymbol{v})}{n+\boldsymbol{v}}\beta + \frac{h}{m_0}\sqrt{1-\beta^2}(n-\boldsymbol{v})\frac{1-\cos\varphi}{(1-\beta)^2} \\
&\approx 0 - T_c\sqrt{\frac{Q-1}{2\ln Q}} - \frac{h}{m_0}\sqrt{\frac{Q-1}{\ln Q}}\frac{1-\cos\varphi}{\left(1-\sqrt{1-\frac{Q-1}{\ln Q}}\right)^2} \\
&\approx -2\pi R\sqrt{\frac{2\ln Q}{Q-1}}
\end{aligned}
\tag{30}
$$

In Eq. (30) the following equation from Ref. [1] is used

$$
\lim_{v\to 1}\frac{1}{\gamma} = \lim_{v\to 1}\sqrt{1-\beta^2} = \lim_{v\to 1}\sqrt{1-\left(\frac{v}{k}\right)^2} = \lim_{v\to 1}\sqrt{\frac{1-v}{n}(n+v)} = \sqrt{\frac{Q-1}{2\ln Q}} \tag{31}
$$

Where the expression for n is shown in Eq. (8).

On the other hand, when $\theta_1 = \pi, \cos\theta_1' = (-1-\beta)/(1+\beta) = -1$, then it means $\theta_1' = \pi$, so $n = n\left(\boldsymbol{v}\cos\left(\pi-\theta_1'\right), \boldsymbol{c}\right) = n(\boldsymbol{v}, \boldsymbol{c})$ in Eq. (17). Then we can obtain

$$
\begin{aligned}
\lim_{\boldsymbol{v}\to 1}\lambda_2 &= \lim_{\boldsymbol{v}\to 1}\frac{n-\boldsymbol{v}}{\sqrt{1-\beta^2}}\left[T_c\frac{nk(1-n\boldsymbol{v})}{n-\boldsymbol{v}}\left(1-\frac{k}{n}\beta\right)\right] - \frac{h}{m_0}\sqrt{1-\beta^2}(n-\boldsymbol{v})\frac{1-\cos\varphi}{(1-\beta)^2} \\
&= \lim_{\boldsymbol{v}\to 1}\frac{T_c}{\sqrt{1-\beta^2}}nk(1-n\boldsymbol{v}) - \frac{T_c}{\sqrt{1-\beta^2}}k^2(1-n\boldsymbol{v})\beta - \frac{h}{m_0}\sqrt{1-\beta^2}(n-\boldsymbol{v})\frac{1-\cos\varphi}{(1-\beta)^2} \\
&= 0 - T_c\sqrt{\frac{2\ln Q}{Q-1}} + \frac{h}{m_0}\sqrt{\frac{Q-1}{\ln Q}}\frac{1-\cos\varphi}{\left(1-\sqrt{1-\frac{Q-1}{\ln Q}}\right)^2} \\
&\approx -2\pi R\sqrt{\frac{2\ln Q}{Q-1}}
\end{aligned}
\tag{32}
$$

In Eq. (30) and Eq. (32) we obtained a negative value because of the timing sequence is reversed due to the effect of shock wave of light as stated in Sect. 2. And it can be observed in Eq. (30) and Eq. (32) that when $v = n, \lambda_2 = 0$ (As shown above, although $\lambda_2 = 0$ in this case, both the momentum and energy of the photon are a limited value), and when $\boldsymbol{v} \in (n, 1), \lambda_2$ increases dramatically and tends to be a limited value (due to $Q \sim 0$), which is obviously different from the results predicted by the Lorentz model that in the case where $\boldsymbol{v} \to \boldsymbol{c}$, the observed wavelength of light by the observer in the laboratory system is close to 0 regardless of the location one observed (correspondingly, the momentum and energy of the photon are infinite).

5. Inverse Compton Scattering

The interaction between photons and other particles is often occurred in high energy field, for example, the interaction between gamma-photons and nuclei, the interaction between gamma-

photons and nucleons, etc. The scattering between extremely relativistic particles and photons is called inverse Compton scattering process, for that photons can obtain energy from particles in this process.

Unlike the Compton scattering process, in which the particles are basically stationary, in the inverse Compton scattering process the particles are extremely relativistic and move at very high speeds. However, the method of deriving the relationship between the scattered photon and the incident photon is the same, that is, they all use the law of energy conservation and momentum conservation shown in Eq. (12). First we can build an "electron-rest reference frame" S' that moves along with the electron (here we analyze the interaction between photons with electrons). In S', the collision between photons and electrons satisfies the Compton's formula as Eq. (18)

$$\frac{\lambda_2'}{a_2'} - \frac{\lambda_1'}{a_2} \approx \frac{h}{m_0}\left(1 - \cos\varphi'\right) \tag{33}$$

Where λ_1' and $\lambda_2{}'$ represent the wavelengths of incident and scattered photons observed in S' respectively, φ' denotes the scattering angle.

Here we assume that both the observer and the light source are in the S inertial system, and that S is moving at a velocity v relative to S', then based on Eq. (6) and Eq. (25) it has

$$\begin{aligned} \lambda_1' &= n\left(1 - \frac{\boldsymbol{v}\cos\theta}{n\boldsymbol{c}}\right)\frac{\lambda_1}{\sqrt{1-\beta^2}} \\ a_2 &= n\frac{1 - \frac{\boldsymbol{v}\cos\theta}{n\boldsymbol{c}}}{1 - n\frac{\boldsymbol{v}\cos\theta}{\boldsymbol{c}}} \\ \lambda_2 &= n\left(1 - \frac{\boldsymbol{v}\cos\theta}{n\boldsymbol{c}}\right)\frac{\lambda_2'}{\sqrt{1-\beta^2}} \end{aligned} \tag{34}$$

Where λ_1 and λ_2 represent the wavelengths of incident and scattered photons observed in S respectively.

Next we consider the case where $\theta = 0$ (for that in this case a maximum energy can be obtained by the photon from the particle), and correspondingly it has $\varphi' = \pi$. Since Eq. (33) describes the Compton scattering that occurs in S', we can assume that $a_2' \approx 1$.

Then based on Eq. (33)∼ (34), the observer in S can observe the scattered photon with momentum $\boldsymbol{P}_2$

$$\boldsymbol{P}_2 = a_2''\frac{h}{\lambda_2} = \frac{n-\boldsymbol{v}}{1-n\boldsymbol{v}}\frac{h}{\frac{n-\boldsymbol{v}}{\sqrt{1-\beta^2}}\left[\frac{2h}{m_0} + (1-n\boldsymbol{v})\frac{\lambda_1}{\sqrt{1-\beta^2}}\right]} = \frac{1}{1-n\boldsymbol{v}}\frac{h}{\frac{1}{\sqrt{1-\beta^2}}\frac{2h}{m_0} + (1-n\boldsymbol{v})\frac{\lambda_1}{1-\beta^2}} \tag{35}$$

When $n \equiv 1$ Eq. (35) returns to the case as in the Lorentz model.

From Eq. (35) it can be seen that when $n = \boldsymbol{v}$, $\boldsymbol{P}_2$ has a maximum value, that is

$$\boldsymbol{P}_2^{\max} = \frac{1}{1-n\boldsymbol{v}}\frac{h}{\frac{1}{\sqrt{\frac{1-\boldsymbol{v}}{n}(n+\boldsymbol{v})}}\frac{2h}{m_0} + (1-n\boldsymbol{v})\frac{\lambda_1}{\frac{1-\boldsymbol{v}}{n}(n+\boldsymbol{v})}} \approx \frac{h}{\frac{4\sqrt{2}h}{m_0}\sqrt{1-\boldsymbol{v}_{\text{wave shock}}} + \lambda_1\left(1-\boldsymbol{v}_{\text{wave shock}}\right)} \tag{36}$$

Where $\boldsymbol{v}_{\text{wave shock}}$ comes from Eq. (9).

For the observer in S, if the observed energy of the electron is much greater than the observed energy of the incident photon, Eq. (36) can be further simplified as

$$\boldsymbol{P}_2^{\max} \approx \frac{\sqrt{2} m_0}{8\sqrt{1-\boldsymbol{v}_{\text{wave shock}}}} \tag{37}$$

And what we're more interested in is that what happens if $\boldsymbol{v} \to \boldsymbol{c}$, in which case $n \to 0$, then based on Eq. (35) we can obtain

$$\lim_{\boldsymbol{v}\to 1} \boldsymbol{P}_2 = \lim_{\boldsymbol{v}\to 1} \frac{h}{\frac{1}{\sqrt{1-\beta^2}} \frac{2h}{m_0} + \frac{\lambda_1}{1-\beta^2}} = \frac{h}{\sqrt{\frac{2\ln Q}{Q-1}} \frac{2h}{m_0} + \frac{2\ln Q}{Q-1}\lambda_1} \tag{38}$$

As one knows, in the Lorentz model, as the velocity of the electron is close to the speed of light, the maximum energy that a photon can obtain through inverse Compton scattering process tends to be infinite. However, if Q is not equal to 0 (when $Q = 0$, then $n \equiv 1$ and the model discussed above will return to the Lorentz model), then the maximum energy that a photon can obtain through inverse Compton scattering process will have a limit, which is shown in Eq. (37), and when further the velocity of electron is high enough to be close to c, the maximum energy that a photon can obtain instead drops to another non-zero and limited value, which is mainly due to the property of n.

In recent years, the mechanism of high energy radiation of blazars have been studied extensively [25–31]. One model used to explain this mechanism is the lepton origin model, which considered that the high energy radiation comes from the inverse Compton scattering process between relativistic electrons and low energy photons [29–31]. Since some special objects with extreme physical properties, such as the blazars, can produce extremely relativistic particle jets, and the microwave background radiation (CMB) provides natural low-energy photons, we are looking forward to testing the value of Q in future experiment on the ultra-high energy radiation, for that whether the value of Q is 0 will have a very different effect on the ultra-high energy radiation spectrum, which is shown in this section.

6. Summary

The Lorentz violation model is widely studied in the ultra-high energy field in recent years, especially is considered in various models attempting to unify the general relativity and quantum theory [32**?** –37]. Among them Ref. [1] proposed a possible Lorentz violation model that the speed of light is relate to the speed of light source and thus may changes between inertial systems. The key idea in Ref. [1] is that in the case where $\boldsymbol{v} \to \boldsymbol{c}$ the energy or momentum of the particle have a limited value (i.e., the time or length scaling factor limit is $\sqrt{2\ln Q/(Q-1)}$, and correspondingly, the energy limit of a particle with rest mass m_0 is $m_0 c^2 \sqrt{2\ln Q/(Q-1)}/[1-0.5(Q-1)/\ln Q]\Big)$ rather than be infinite predicted by the Lorentz model. The model in Ref. [1] is similar to the famous rainbow model, but they are different, for that in Ref. [1] the energy limit of the particle is related to the rest mass of the particle, while in the rainbow model it is not, but is assumed to be a constant (the constant is usually considered to be the Planck energy or near the Planck energy). If we follow the viewpoint in Ref. [1] that the speed of light observed by an observer moving with velocity $\boldsymbol{v}$ relative to the light source is $n\boldsymbol{c}$, then it will forces us to have to modify the usual quantum relation of photons involved in the Doppler effect.

By discussing the Doppler effect of photons based on the idea in Ref. [1], we found that light can also have shock phenomena (in vacuum) just like sound wave, and further we obtained the conditions for shock wave of light, that is, $\boldsymbol{v} = n\boldsymbol{c}$, and in the case where $n\boldsymbol{c} < \boldsymbol{v} < \boldsymbol{c}$ the timing sequence will be reverse, but it doesn't violate the law of causality for that the timing sequence can be reversed in independent events (but not in dependent events). Then we discussed the Compton scattering process between the photon with the modified quantum relation as shown in Eq. (11) and the electron, and found that unless we define a suitable correction coefficient for the quantum relation of photons, the photons energy will be infinite in the case where $\boldsymbol{v} = n\boldsymbol{c}$, which makes it impossible to discuss the Compton scattering process between photons and electrons over the whole range of $\boldsymbol{v} \in [0, \boldsymbol{c})$, in other words, because of the infinite energy of photon in the case where $\boldsymbol{v} = n\boldsymbol{c}$, the Compton scattering between photons and electrons will be meaningless. So in order to avoid that we defined a suitable correction coefficient for the quantum relation of photons to make the energy of photons have a limited value over the whole range of $\boldsymbol{v} \in [0, \boldsymbol{c})$, especially in the case where $\boldsymbol{v} = n\boldsymbol{c}$ or $\boldsymbol{v} \to \boldsymbol{c}$.

Here it should be noted that, due to the current limited information or knowledge we know, we are not yet able to obtain the specific correction coefficients for photon's quantum relation satisfying Eq. (2), but we can obtain three key restrictive conditions for the possible correction coefficients, which is shown in Sect. 4. One may propose a lot of expressions for the correction coefficients satisfying the three restrictive conditions, but here in this paper after many attempts we construct a very concise expression for the correction coefficients, which as shown in Eq. (25).

Then based on Eq. (25) we studied the spontaneous radiation in a cyclotron maser, for that the phenomenon can be explained by the concept of virtual photon. By analyzing the Doppler effect and Compton scattering effect of virtual or real photons, we obtained the wavelength of spontaneous radiation and synchrotron radiation in the cyclotron maser. The important conclusion is that we found in the case where $\boldsymbol{v} \to \boldsymbol{c}$ the wavelength observed by the observer in the laboratory system does not tend to be 0, which is derived from the Lorentz model and which means the energy and momentum of photon tend to be infinite, but tends to be a limited value. And even if in the case where $\boldsymbol{v} = n\boldsymbol{c}$, the wavelength is 0 , but the energy and momentum of photon are both a limited value, which ensures the Compton scattering is meaningful over the whole range of $\boldsymbol{v} \in [0, \boldsymbol{c})$, since in Ref. [1] it allows $\boldsymbol{v} \in [0, \boldsymbol{c})$.

Further, we also discussed the inverse Compton scattering phenomenon based on Eq. (25), and found that there is a limit to the maximum energy that can be obtained by photons in the collision process between ultra-high energy particles and low-energy photons, and this case occurs when the particle's moving speed satisfy $\boldsymbol{v} = \boldsymbol{v}_{\text{wave shock.}}$. When the velocity of particle is greater than $\boldsymbol{v}_{\text{wave shock}}$ and close to $\boldsymbol{c}$, the energy that can be obtained by the photon decreases sharply and tends to be another limited value. This conclusion is also very different from that predicted by the Lorentz model, in which the energy that can be obtained by the photon tends to be infinite as the velocity of particle is close to $\boldsymbol{c}$.

In general, the basic idea of this paper is the same as that of Ref. [1], that is, to avoid the energy or momentum of particles (i.e., any particles, including photons) to be infinite, otherwise it will make the real physical situation meaningless in some cases, and which is also inspired by the idea of some quantum gravity models [34–37]. It is very important to emphasize that if $Q = 0$, then $n \equiv 1$ and correspondingly, all the results and conclusions in this paper will return to the case as in the Lorentz model. So this paper also provides us with a possible experimental scheme to determine the value of Q, although it still requires extremely high experimental energy.

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